Stationary Online Contention Resolution Schemes

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Online resource allocation problems have been widely studied in the revenue management literature. In these problems, a seller with limited inventory adaptively decides whether or not to fulfill each stochastically arriving request to maximize the total expected revenue collected. A flexible method to solve such problems is to first solve a fluid approximation of the problem, and then find ways to "round" such a solution into a feasible algorithm for allocating resources in an online fashion. Online contention resolution schemes (OCRS) are an important tool in this methodology. They provide allocation policies that balance the fulfillment probability (conditional on a request being made) among all potential buyers. Along with being technically useful, the fairness guarantee that is intrinsic in their definition is extremely relevant for societal applications, where a fair allocation among users is highly desired, if not mandated.

However, optimal and near-optimal OCRSs are often hard to design and interpret. In this paper, we introduce new conceptual and technical paradigms that aim to simplify the design, analysis, and interpretation of OCRSs. In particular, we define the notion of a *"stationary" online contention resolution scheme (S-OCRS)*. Such a scheme aims to treat every request it receives as if they were the very last one, and makes fictional allocations in order to ensure that the "perceived" level of inventory remains stationary in distribution over time. We develop a simple framework for designing and analyzing these schemes.

We also introduce a new technical paradigm for the design and analysis of OCRSs, where we consider a surrogate problem in which algorithms are allowed to serve a single request multiple times. This proxy model arises from the analysis of the so-called "vanishing case" of contention resolution scheme. Then, we introduce a novel reduction to show how we can use such algorithms in the proxy model to design OCRSs that serve each request at most once, ensuring that the inventory distribution of the derived policy is always a mean-preserving-contraction (MPC) of that of an algorithm serving requests multiple times.

We give several applications of our new perspectives on contention resolution schemes. Firstly, we explain how stationary contention resolution schemes can be applied to design algorithms for revenue management problems beyond the traditional, online arrival setting. Secondly, focusing on the special case of contention resolution schemes for an inventory of size k, we design contention resolution schemes that provide near-optimal, interpretable, greedy-like S-OCRSs. We combine this with our MPC paradigm to design an OCRS with a near optimal selectability of $1 - 2/\sqrt{2\pi k} + O(1/k)$. Lastly, our MPC framework also provides a new proof of why a greedy algorithm based on the level of available inventory is the optimal policy, which is arguably simpler and more intuitive than the previous proofs using linear programs.